Drag Reduction in External Rotational Flows

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The correlation of drag reduction data proposed by Astarita et al. (1969) (AGN correlation) is rather appealing due to its simplicity. Patterson et al. (1970) compared the shape of the AGN correlation with that of a correlation based on a model with some phenomenological considerations. Peterson and Beckwith (1971) have examined the validity of AGN correlation for systems of polymers dissolved in polar and nonpolar solvents. In this work some interesting features of this type of correlation are examined with respect to drag reduction in certain external flow problems. Some limitations of this type of correlation are also pointed out.

In particular, the problem of correlation of turbulent drag reduction data for flow around rotating discs, cones, and cylinders is considered. It is generally agreed that drag reduction arises out of some interaction of turbulence and elasticity, although the exact manner of such an interaction is not completely clear. The data could be usually correlated by assuming that the friction factor (or power number) is a function of the Reynolds number *Re* and a Deborah number *De*. The latter corresponds to a ratio of the natural time of the fluid *T* and a characteristic process time *t*.

The choice of the characteristic time of the process is primarily based on the postulated mechanism of the phenomenon of drag reduction. Kelkar and Mashelkar (1972) have, however, shown recently that in spite of the apparent diversity of the postulated mechanisms of drag reduction, the characteristic process time has remarkably the same form. Thus, for instance in the case of pipe flow the mechanisms of drag reduction proposed by Hershey and Zakin (1967), Seyer and Metzner (1967), Meek and Baer (1970) and Gordon (1970) are fundamentally quite different but their arguments result approximately in the same form of characteristic process time. The simplest model for drag reduction under external flow conditions will be chosen in this work. This model is based on the ideas extended by Astarita (1965), which make use of the arguments of Levich (1962).

Under turbulent flow conditions a rotating body continuously generates eddies determined by certain sizes and frequency distribution and also by certain geometric orientation. The large scale eddies are of the size of the characteristic dimension of the rotating body and have their periods equal to the reciprocal of the angular velocity of the body. These large scale eddies transfer the kinetic energy to smaller and smaller eddies and continuation of this process eventually leads to eddies so small that viscous dissipation is very rapid. These small scale high frequency dissipating eddies are in the universal equilibrium range, which is independent of the geometric details of the body and is only dependent upon the rate at which the energy is supplied and the tendency of these eddies to dissipate this energy viscously. In other words, the properties of the small scale eddies are determined by the transfer processes and not by the properties of the mechanism of turbulence generation. Using the arguments extended by Levich (1962) the frequency of these dissipating eddies can be found as

$$\nu = \omega \, Re^{0.5} \tag{1}$$

For most practical rotational flow situations the reciprocal frequencies are of the same order of magnitude as the fluid relaxation time of the dilute polymer solutions (10^{-2} to 10^{-5} s). Under such conditions the dissipative frequency range of the energy spectrums will become conservative and drag reduction will occur, a situation quite similar to the case of a Maxwell model stretched at frequencies higher than the reciprocal relaxation time. The proper form of Deborah number to be considered with this line of thought is

$$De = \omega \, Re^{0.5} \, T \tag{2}$$

Although there is some doubt as to whether turbulence becomes less dissipative as assumed in this model or whether there is a reduced generation of turbulence as assumed by some other workers [for example, Walsh (1967), Gadd (1965)], the details of the exact mechanism are unlikely to affect the final correlation of the drag reduction data substantially.

Following the method of AGN correlation, we assume that the moment coefficient, C_M for the rotating body is a function of *Re* and *De*. The Reynolds number dependence is then eliminated by defining a dimensionless moment coefficient as

$$\frac{C_M}{C_{Mo}} = \beta(De) \tag{3}$$

where C_{Mo} is the power number for a purely viscous fluid. The fluid relaxation time *T* is now replaced by a number proportional to the value of the reciprocal frequency ν at $\beta = 0.7$, referred to as $\nu_{0.7}$. This gives an alternative functional form

$$\frac{C_M}{C_{Mo}} = \beta(\nu/\nu_{0.7}) \tag{4}$$

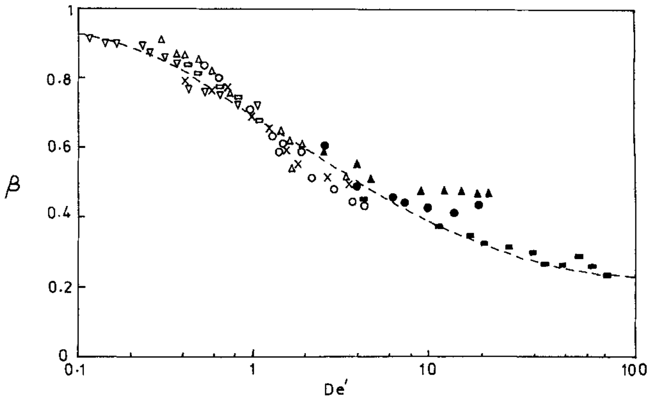

Fig. 1. The curve showing the functional dependence of the reduced power number (or friction factor), β on the modified Deborah number, De'. ------- straight and coiled tube curve of Kelkar and Mashelkar; ▽ 10 ppm Guargum (disk); □ 30 ppm Guargum (disk); △ 60 ppm Guargum (disk); × 100 ppm Guargum (disk); ○ 300 ppm Guargum (disk); ▲ 100 ppm Polyox (cone); ● 500 ppm Polyox (cone); and ■ 50 ppm Polyox (cylinder).

The data are preferably correlated with $\nu_{0.7}$ rather than $\nu_{0.5}$ as in AGN correlation because it was difficult to find data where reductions as large as 50% were obtained consistently. The preference for using $\nu_{0.8}$ has been shown by Gordon (1971) for similar reasons.

The data in the literature are now analyzed for establishing the exact functional form of the correlation expressed in Equation (4). The drag reduction data reported by Bilgen (1971) on turbulent flows around a disk (25.4 cm diam.) in aqueous Guar gum solutions (10 to 300 ppm) were analyzed. Also analyzed were the data by Killen and Almo (1969) on a cone (30.48 cm diam. with 45° apex angle) and a cylinder (30.48 cm diam. and 30.48 cm effective length) rotating in Polyox (WSR 301) solutions (50 to 500 ppm). The Reynolds number in all the cases was defined as $Re = R^2\omega\rho/\mu$ and the definitions of the moment coefficient were chosen in accordance with the usual practice (see Notation). The C_M-Re curves for purely viscous liquids (Newtonian for the solutions considered) were taken from standard references (Goldstein 1935; Theodorsen and Regier, 1944). Wherever possible, the data obtained by the above authors with water, were used.

The plot $\beta(\nu/\nu_{0.7})$ is shown in Figure 1. It is satisfying to see that the data are correlated by a unique curve over a range of $\nu/\nu_{0.7}$ of about three orders of magnitude.

Kelkar and Mashelkar (1972) have analyzed extensive data on drag reduction in dilute aqueous polymer solutions in straight pipes. They have also obtained experimental data on turbulent drag reduction in aqueous polyacrylamide solutions in a helically coiled tube (under the conditions of an intense secondary flow called a double helical flow). All these data were satisfactorily correlated by a single expression

$$\beta = 0.2 + \frac{0.8}{1 + 0.6(\nu/\nu_{0.7})^{0.83}} \tag{5}$$

where ν has been appropriately defined for the straight and the coiled pipe. Figure 1 shows Equation (5) plotted by a dotted line. A surprisingly good agreement is found between the internal and external flow correlations. This interesting feature of a unique correlation of drag reduction data for a range of internal and external flow conditions appears to be of a great potential importance because it indicates the possibility of obtaining a complete information about drag reduction in different flow situations by using relatively limited experimental data. This conclusion will, however, have to be confirmed with extensive and reliable data on external flow drag reduction.

The only previous effort to correlate external flow drag reduction data was made by Peyser and Little (1971), who used a Deborah number defined as $De = \omega T$. Taking ω as the characteristic process time is equivalent to taking it the same as the time scale of the large scale nondissipating eddies, which are relatively unimportant in the process of energy dissipation. This choice hence sounds physically unrealistic. It is interesting to find that for obtaining a satisfactory correlation Peyser and Little found it necessary to modify the above De as $De = \omega f(Re)T$, the function $f(Re)$ varied approximately as $Re^{0.4}$ in the range used. This indirectly confirms the analysis of this work.

One of the drawbacks of AGN type of correlation is that for a given concentration (fixed $\nu_{0.7}$) it predicts that with a sufficient increase in ν (or Re), it should be possible to attain an asymptotic value of $\beta = 0.2$. The data in the literature contradicts this. An analysis of the data by Wang (1969) shows that for $Re > 10^5$, β versus $\nu/\nu_{0.7}$ curves gradually attain a constant value of β which is different for different concentrations. Not many studies in the literature have reported drag reduction data with Re much greater than 10^5. However, AGN correlation is obviously applicable only before these asymptotic regions are reached. Some of the data in Figure 1 (see for example, 100 and 500 ppm Polyox data on cones) show such asymptotes as well. This limitation will hence have to be borne in mind before using AGN type of correlation.

NOTATION

C_M = moment coefficient for the polymer solution (= $M/\rho\omega^2R^5$ for disc, = $M/\rho\omega^2R^4l$ for cylinder, = $M\sin\theta/\rho\omega^2R^5$ for cone)
C_{Mo} = moment coefficient for solvent
De = Deborah number
De' = $\nu/\nu_{0.7}$ = modified Deborah number
l = effective length of the cylinder, cm
M = moment, dyne cm
R = radius, cm
Re = Reynolds number
t = a characteristic process time, s
T = natural time of the fluid, s
β = reduction ratio
θ = apex angle, radians
ν = characteristic frequency, s^{-1}
$\nu_{0.7}$ = value of ν for $\beta = 0.7$, s^{-1}
ρ = density, gm/cm^3
ω = angular velocity, s^{-1}

LITERATURE CITED

Astarita, G., "Possible Interpretation of the Mechanism of Drag Reduction in Viscoelastic Liquids," *Ind. Eng. Chem. Fundamentals*, **4**, 354 (1965).

———., G. Greco, and L. Nicodemo, "A Phenomenological Interpretation and Correlation of Drag Reduction," *AIChE J.*, **15**, 564 (1969).

Bilgen, E. A., "On the Stability of Viscoelastic Flow due to a Rotating Disk," *Chem. Eng. Progr. Ser. No. 111*, **67**, 74 (1971).

Gadd, G. E., "Turbulence Damping and Drag Reduction Produced by Certain Additives in Water," *Nature*, **206**, 463 (1965).

Goldstein, S., "On the Resistance to the Rotation of a Disc Immersed in a Fluid," *Proc. Cambridge Phil. Soc.*, **31**, 232 (1935).

Gordon, R. J., "On the Explanation and Correlation of Turbulent Drag Reduction in Dilute Macromolecular Solutions," *J. Appl. Polym. Sci.*, **14**, 2097 (1970).

———., "Comments on "A Phenomenological Interpretation and Correlation of Drag Reduction," *AIChE J.*, **17**, 1013 (1971).

Hershey, H. C., and J. L. Zakin, "A Molecular Approach to Predicting the Onset of Drag Reduction in the Turbulent Flow of Dilute Polymer Solutions," *Chem. Eng. Sci.*, **22**, 1847 (1967).

Kelkar, J. V., and R. A. Mashelkar, "Drag Reduction in Dilute Polymer Solutions," *J. Appl. Polym. Sci.*, in press.

Killen, J. M., and J. Almo, "An Experimental Study of The Effects of Dilute Solutions of Polymer Additives on Boundary Layer Characteristics," in *Viscous Drag Reduction*, C. S. Wells (ed.), Plenum Press, N. Y. (1969).

Levich, V. G., *Physico-Chemical Hydrodynamics*, p. 20, Prentice-Hall, Englewood Cliffs, N. J. (1962).

Meek, R. L., and A. D. Baer, 'The Periodic Viscous Sublayer Model and Heat Transfer to Drag Reducing Solutions," *AIChE J.*, **16**, 1100 (1970).

Patterson, G. K., J. L. Zakin, and J. M. Rodriguez, "Drag Reduction Correlations," *ibid.*, 505.

Peterson, J. P., and W. F. Beckwith, "A Communication on the Drag Reduction Correlation proposed by Astarita, Greco and Nicodemo," *ibid.*, **17**, 1505 (1971).

Peyser, P., and R. C. Little, "The Drag Reduction of Dilute

Polymer Solutions as a Function of Solvent Power, Viscosity and Temperature," *J. Appl. Polym. Sci.*, **15**, 2623 (1971).

Seyer, F. A., and A. B. Metzner, "Turbulent Flow Properties of Viscoelastic Fluids," *Can. J. Chem. Eng.*, **45**, 121 (1967).

Theodorsen, T., and A. Regier, "Experiments on Drag of Revolving Discs, Cylinders and Streamline Rods at High Speed," NACA Report 793 (1944).

Walsh, M., "Theory of Drag Reduction in Dilute High Polymer Flows," *Intern. Shipbldg. Progr.*, **14**, 134 (1967).

Wang, C. B., "Pipe Flow of Dilute Polymer Solutions," Ph.D. thesis, Univ. Wisconsin, Madison (1969).

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Interfacial Turbulence at the Surface of a Drop in a Liquid-Liquid Reaction System

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Studies of interfacial phenomena in two phase liquid-liquid reaction systems have indicated an enhancement in the rate of the chemical reaction due to the presence of interfacial turbulence. The nature and the magnitude of interfacial turbulence depends upon the systems under consideration, temperature, and the direction for mass transfer.

For the case of mass transfer without chemical reaction (Bakker et al., 1966, 1967), convection cells are formed on the surface of a suspended drop when the mass transfer of solute is toward the drop. Internal and external circulation is observed when the mass transfer is away from the drop. Treating the problem of interfacial turbulence as one of Marangoni instability, Scriven and Sternling (1959) showed that if the interfacial tension decreases with increasing solute concentration, instability occurs whenever the solute is transferred out of the phase of lower solute diffusivity. Presence of instability in this analysis indicated the presence of interfacial turbulence in real systems.

We attempt here an order of magnitude estimation of the scale and velocity of convection cells due to interfacial turbulence and their contribution to the rate process.

EXPERIMENTAL APPARATUS

The optical system used to observe interfacial turbulence consisted of an optical bench, a prefocused lamp, an adjustable slit, a 10X objective, a corresponding eyepiece and a Bauch and Lomb L photomicrographic camera mounted horizontally. The camera was equipped with a 3¼-in. by 4¼-in. ground glass screen for visual observation and a cut film holder for taking pictures. For most of the photographs, shutter speeds of 1/25 and 1/50 seconds were used.

Reactions were carried out in a 10 × 10 × 80 mm optical cell placed inside a 20 × 30 × 60 mm optical cell on a support grid. The outer cell was provided with an inlet and outlet for circulating water at a predetermined temperature from a water bath. A Gilmont microburet 0.2 ml in capacity and accurate to 0.0002 ml was used to suspend the drops. The tip of the microburet was carefully treated with a dilute solution of Siliclad to make it nonwetting to aqueous solutions and obtain nearly spherical drops.

Before every run, the microburet was taken apart, washed in distilled water, dried, and reassembled. Reactant A was sucked into the buret and the continuous phase reactant B was placed in the inner cell. Water was circulated in the outer cell. About 30 minutes were allowed for achieving steady state in temperature, and then a drop was carefully formed at the tip of the microburet. It was observed on the ground glass screen. Several times during the course of a reaction, the ground glass screen was replaced by a cut-film holder and photographs were taken. Photographs were enlarged to an 8 × 10-in. print size before analysis. Many times the negative was projected on a screen for counting and averaging cell areas. On the print a cell of a 1 micron occupied 0.3 mm-1.0 mm depending on the degree of enlargement. Accuracy of cell size measurement was limited to 0.5 microns. The velocity of the cells was measured by timing movements of individual cell image on the surface of the drop. Points on Figures 1 and 2 represent average values of a number of observations which followed a single cell about 2 cm for about 2 seconds. The accuracy of the velocity was limited to 0.5 microns/sec. Scriven's (1961) criteria was used to determine the direction of the reaction zone movement. In all cases considered here the reaction zone had moved into the dispersed drop phase.

Reactants Used

Two reaction systems were used. In the first pure ethyl acetate constituted the organic continuous phase and dilute NaOH (in the concentration range 0.08 to 0.3N) was the dispersed phase. The second system consisted of a 50% solution of the toluene in DiEthylAmine (DEA) as the continuous organic phase and a dilute solution of acetic acid (25% to 45% by weight) as the dispersed phase.